Guaranteed Transient Performance with $\mathcal{L}_1$ Adaptive Controller for Systems with Unknown Time-varying Parameters: Part I *

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Abstract

This paper presents a novel adaptive control methodology for uncertain systems with time-varying unknown parameters and time-varying bounded disturbance. The adaptive controller ensures uniformly bounded transient and asymptotic tracking for system's both signals, input and output, simultaneously. The performance bounds can be systematically improved by increasing the adaptation gain. Simulations of a robotic arm with time-varying friction verify the theoretical findings.

1 Introduction

This paper presents an adaptive control methodology for controlling systems with unknown time-varying parameters, which are not required to have slow variation. The methodology ensures uniformly bounded transient response for system's both signals, input and output, simultaneously, in addition to asymptotic tracking. The main advantage of this new architecture, as compared to the existing results in the literature, is that it ensures uniform transient tracking for system's input signal in addition to its output. The $\mathcal{L}_\infty$ norm bounds for the error signals between the closed-loop adaptive system and the closed-loop reference LTI system can be systematically reduced by increasing the adaptation gain.

Adaptive algorithms achieving arbitrarily improved transient performance in case of constant unknown parameters are given in [1–12], and for unknown time-varying parameters have been given in [13, 14]. While the results in [13, 14] improved upon [15–17], by extending the class of systems beyond the slow time-

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variation of the unknown parameters and guaranteeing performance improvement to arbitrary degree, they still did not provide means for regulating the performance of the control signal during the transient.

A common tendency observed in a variety of applications using adaptive control is that increasing the adaptation gain leads to improved transient tracking of the system output, but the control signal experiences high-frequency oscillations. In [18], a bound is derived to confirm the first part of this statement assuming appropriate trajectory initialization. The high-frequency oscillations in the control signal consequently limit the rate of adaptation. If one considers the simplest adaptive scheme for a scalar linear system with constant disturbance, which can be solved by a PI controller, then it is straightforward to verify that increasing the adaptation gain leads to reduced phase margin for the resulting closed-loop linear system, [19]. This observation explains to some extent the oscillations inherent to the control signal in the presence of high adaptation gain.

In recent papers [20, 21], we have developed a novel $\mathcal{L}_1$ adaptive control architecture that permits fast adaptation and yields guaranteed transient response for system's both signals, input and output, simultaneously, in addition to asymptotic tracking. The main feature of it is the ability of fast adaptation with guaranteed low-frequency control signal. The ability of fast adaptation ensures the desired transient performance for system's both signals, input and output, simultaneously, while the low-pass filter in the feedback loop attenuates the high-frequency components in the control signal. In this paper we expand the class of systems to have time-varying unknown parameters of arbitrary rate of variation, and we correspondingly modify the architecture from [20, 21] to ensure the desired transient performance for system's both signals. We prove that by increasing the adaptation gain one can achieve arbitrary close transient and asymptotic tracking for system's both signals, input and output, simultaneously. In Part II of this paper [22], we prove that increasing the adaptation gain will not hurt the time-delay margin of the closed-loop system with the $\mathcal{L}_1$ adaptive control architecture, as opposed to the conventional adaptive schemes observed in [19].

The paper is organized as follows. Section 2 states some preliminary definitions, and Section 3 gives the problem formulation. In Section 4, the novel $\mathcal{L}_1$ adaptive control architecture is presented. Stability and uniform transient tracking bounds of the $\mathcal{L}_1$ adaptive controller are presented in Section 5. In section 6, simulation results are presented, while Section 7 concludes the paper.

2 Preliminaries

In this Section, we recall some basic definitions and facts from linear systems theory, [23–25].

Definition 1: For a signal $\xi(t),\ t \geq 0,\ \xi \in \mathbb{R}^n$, its truncated $\mathcal{L}_\infty$ norm and $\mathcal{L}_\infty$ norm are defined as

$$\begin{aligned} \|\xi_t\|_{\mathcal{L}_\infty} &= \max_{i=1,..,n} \left(\sup_{0\leq\tau\leq t} |\xi_i(\tau)| \right), \\ \|\xi\|_{\mathcal{L}_\infty} &= \max_{i=1,..,n} \left(\sup_{\tau\geq 0} |\xi_i(\tau)| \right), \end{aligned}$$

where ξ_i is the i^{th} component of ξ.

Definition 2: The $\mathcal{L}_1$ gain of a stable proper single–input single–output system $H(s)$ is defined to be $||H(s)||_{\mathcal{L}_1} = \int_0^\infty |h(t)|dt$, where $h(t)$ is the impulse response of $H(s)$, computed via the inverse Laplace transform $h(t) = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} H(s)e^{st}ds, t \geq 0$, in which the integration is done along the vertical line $x = \alpha > 0$ in the complex plane.

Proposition: A continuous time LTI system (proper) with impulse response $h(t)$ is stable if and only if $\int_0^\infty |h(\tau)|d\tau < \infty$. A proof can be found in [23] (page 81, Theorem 3.3.2).

Definition 3: For a stable proper m input n output system $H(s)$ its $\mathcal{L}_1$ gain is defined as

$$\|H(s)\|_{\mathcal{L}_1} = \max_{i=1,\cdots,n} \left(\sum_{j=1}^{m} \|H_{ij}(s)\|_{\mathcal{L}_1} \right), \tag{1}$$

where $H_{ij}(s)$ is the i^{th} row j^{th} column element of $H(s)$.

The next lemma extends the results of Example 5.2 ([24], page 199) to general multiple input multiple output systems.

Lemma 1: For a stable proper multi-input multi-output (MIMO) system $H(s)$ with input $r(t) \in \mathbb{R}^m$ and output $x(t) \in \mathbb{R}^n$, we have

$$\|x_t\|_{\mathcal{L}_\infty} \leq \|H\|_{\mathcal{L}_1}\|r_t\|_{\mathcal{L}_\infty}, \quad \forall\, t > 0.$$

Corollary 1: For a stable proper MIMO system $H(s)$, if the input $r(t) \in \mathbb{R}^m$ is bounded, then the output $x(t) \in \mathbb{R}^n$ is also bounded as $\|x\|_{\mathcal{L}_\infty} \leq \|H(s)\|_{\mathcal{L}_1}\|r\|_{\mathcal{L}_\infty}$.

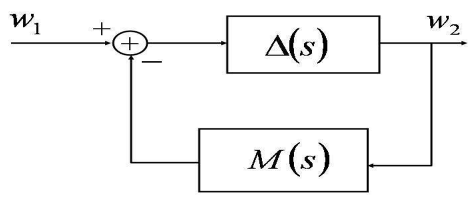


Fig. 1: Interconnected systems

Lemma 2: For a cascaded system $H(s) = H_2(s)H_1(s)$, where $H_1(s)$ is a stable proper system with m inputs and l outputs and $H_2(s)$ is a stable proper system with l inputs and n outputs, we have $\|H(s)\|_{\mathcal{L}_1} \le \|H_2(s)\|_{\mathcal{L}_1}\|H_1(s)\|_{\mathcal{L}_1}$.

Consider an interconnected LTI system in Fig. 1, where $w_1 \in \mathbb{R}^{n_1}$, $w_2 \in \mathbb{R}^{n_2}$, $M(s)$ is a stable proper system with n_2 inputs and n_1 outputs, and $\Delta(s)$ is a stable proper system with n_1 inputs and n_2 outputs.

Theorem 1: ($\mathcal{L}_1$ **Small Gain Theorem**) The interconnected system in Fig. 1 is stable if $\|M(s)\|_{\mathcal{L}_1}\|\Delta(s)\|_{\mathcal{L}_1} < 1$.

The proof follows from Theorem 5.6 ([24], p. 218), written for $\mathcal{L}_1$ gain.

Consider a linear time invariant system:

$$\dot{x}(t) = Ax(t) + bu(t)\,, \tag{2}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, $b \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$ is Hurwitz, and assume that the transfer function $(sI - A)^{-1}b$ is strictly proper and stable. Notice that it can be expressed as:

$$(sI - A)^{-1}b = \frac{n(s)}{d(s)}\,, \tag{3}$$

where $d(s) = \det(sI - A)$ is a n^{th} order stable polynomial, and $n(s)$ is a $n \times 1$ vector with its i^{th} element being a polynomial function:

$$n_i(s) = \sum_{j=1}^{n} n_{ij}s^{j-1}\,. \tag{4}$$

Lemma 3: If $(A \in \mathbb{R}^{n\times n}, b \in \mathbb{R}^n)$ is controllable, the matrix N with its i^{th} row j^{th} column entry n_{ij} is full rank.

Lemma 4: If (A, b) is controllable and $(sI - A)^{-1}b$ is strictly proper and stable, there exists $c \in \mathbb{R}^n$ such that the transfer function $c^\top(sI - A)^{-1}b$ is minimum phase with relative degree one, i.e. all its zeros are located in the left half plane, and its denominator is one order larger than its numerator.

3 Problem Formulation

Consider the following system dynamics:

$$\begin{aligned} \dot{x}(t) &= A_m x(t) + b\left(\omega u(t) + \theta^\top(t)x(t) + \sigma(t)\right), \\ y(t) &= c^\top x(t), \quad x(0) = x_0, \end{aligned} \tag{5}$$

where $x \in \mathbb{R}^n$ is the system state vector (measurable), $u \in \mathbb{R}$ is the control signal, $y \in \mathbb{R}$ is the regulated output, $b, c \in \mathbb{R}^n$ are known constant vectors, A_m is a known $n \times n$ matrix, $\omega \in \mathbb{R}$ is an unknown constant with known sign, $\theta(t) \in \mathbb{R}^{\mathrm{n}}$ is a vector of time-varying unknown parameters, while $\sigma(t) \in \mathbb{R}$ is a time-varying disturbance. Without loss of generality, we assume that

$$\omega \in \Omega = [\omega_l,\ \omega_u], \theta(t) \in \Theta,\ |\sigma(t)| \leq \Delta, \quad t \geq 0, \tag{6}$$

where $\omega_u > \omega_l > 0$ are given bounds, Θ is known compact set and $\Delta \in \mathbb{R}^+$ is a known (conservative) $\mathcal{L}_\infty$ bound of $\sigma(t)$.

The control objective is to design a full-state feedback adaptive controller to ensure that $y(t)$ tracks a given bounded reference signal $r(t)$ *both in transient and steady state*, while all other error signals remain bounded.

We further assume that $\theta(t)$ and $\sigma(t)$ are continuously differentiable and their derivatives are uniformly bounded:

$$\|\dot{\theta}(t)\|_2 \leq d_\theta < \infty, \quad |\dot{\sigma}(t)| \leq d_\sigma < \infty, \quad \forall\, t \geq 0, \tag{7}$$

where $\|\cdot\|_2$ denotes the 2-norm, while the numbers d_θ, d_σ can be arbitrarily large.

4 $\mathcal{L}_1$ Adaptive Controller

In this section, we develop a novel adaptive control architecture for the system in (5) that permits complete transient characterization for both $u(t)$ and $x(t)$. The elements of $\mathcal{L}_1$ adaptive controller are introduced next:

Companion Model: We consider the following companion model:

$$\begin{aligned} \dot{\hat{x}}(t) &= A_m \hat{x}(t) + b\left(\hat{\omega}(t)u(t) + \hat{\theta}^\top(t)x(t) + \hat{\sigma}(t)\right), \\ \hat{y}(t) &= c^\top \hat{x}(t), \quad \hat{x}(0) = x_0, \end{aligned} \tag{8}$$

which has the same structure as the system in (5). The only difference is that the unknown parameters $\omega, \theta(t), \sigma(t)$ are replaced by their adaptive estimates $\hat{\omega}(t), \hat{\theta}(t), \hat{\sigma}(t)$ that are governed by the following adaptation laws.

Adaptive Laws: Adaptive estimates are given by:

$$\begin{aligned}
\dot{\hat{\theta}}(t) &= \Gamma_\theta \mathrm{Proj}(-x(t)\tilde{x}^\top(t)Pb, \hat{\theta}(t)),\ \hat{\theta}(0) = \hat{\theta}_0 && (9)\\
\dot{\hat{\sigma}}(t) &= \Gamma_\sigma \mathrm{Proj}(-\tilde{x}^\top(t)Pb, \hat{\sigma}(t)), \quad \hat{\sigma}(0) = \hat{\sigma}_0 && (10)\\
\dot{\hat{\omega}}(t) &= \Gamma_\omega \mathrm{Proj}(-\tilde{x}^\top(t)Pbu(t), \hat{\omega}(t)), \hat{\omega}(0) = \hat{\omega}_0 && (11)
\end{aligned}$$

where $\tilde{x}(t) = \hat{x}(t) - x(t)$ is the error signal between the state of the system and the companion model, $\Gamma_\theta = \Gamma_c I_{n\times n} \in \mathbb{R}^{n\times n}$, $\Gamma_\sigma = \Gamma_\omega = \Gamma_c$ are adaptation gains with $\Gamma_c \in \mathbb{R}^+$, and P is the solution of the algebraic equation $A_m^\top P + PA_m = -Q, Q > 0$.

Control Law: The control signal is generated through gain feedback of the following system:

$$\begin{aligned}
\chi(s) &= D(s)r_u(s)\,,\\
u(s) &= -k\chi(s)\,, && (12)
\end{aligned}$$

where $r_u(s)$ is the Laplace transformation of $r_u(t) = \hat{\omega}(t)u(t) + \bar{r}(t)$,

$$\bar{r}(t) = \hat{\theta}^\top(t)x(t) + \hat{\sigma}(t) - k_g r(t), \tag{13}$$

$$k_g = -\frac{1}{c^\top A_m^{-1} b}\,, \tag{14}$$

$k \in \mathbb{R}^+$ is a feedback gain, while $D(s)$ is any transfer function that leads to strictly proper stable

$$C(s) = \frac{\omega k D(s)}{1 + \omega k D(s)} \tag{15}$$

with low-pass gain $C(0) = 1$. One simple choice is

$$D(s) = \frac{1}{s}\,, \tag{16}$$

which yields a first order strictly proper $C(s)$ in the following form:

$$C(s) = \frac{\omega k}{s + \omega k}\,. \tag{17}$$

Further, let

$$L = \max_{\theta(t)\in\Theta} \sum_{i=1}^{n} |\theta_i(t)|\,, \tag{18}$$

where $\theta_i(t)$ is the i^{th} element of $\theta(t)$, Θ is the compact set defined in (6). We now state the $\mathcal{L}_1$ performance requirement that ensures stability of the entire system and desired transient performance, as discussed later in Section 5.

$\mathcal{L}_1$**-gain stability requirement:** Design $D(s)$ to ensure that

$$\|G(s)\|_{\mathcal{L}_1} L < 1\,, \tag{19}$$

where $G(s) = (sI - A_m)^{-1} b(1 - C(s))$.

The complete $\mathcal{L}_1$ adaptive controller consists of (8), (9)-(11) and (12) subject to $\mathcal{L}_1$-gain stability requirement in (19). The closed-loop system is illustrated in Fig. 2.

Fig. 2: Closed-loop system with $\mathcal{L}_1$ adaptive controller

In case of constant $\theta(t)$, the stability requirement of the $\mathcal{L}_1$ adaptive controller can be simplified. For the specific choice of $D(s)$ and $C(s)$ in (16) and (17), the stability requirement of $\mathcal{L}_1$ adaptive controller is reduced to

$$A_g = \left[\begin{array}{cc} A_m + b\theta^\top & b\omega \\ -k\theta^\top & -k\omega \end{array} \right] \tag{20}$$

being Hurwitz for all $\theta \in \Theta$, $\omega \in \Omega$.

5 Analysis of $\mathcal{L}_1$ Adaptive Controller

5.1 Closed-loop Reference System

We now consider the following closed-loop LTI reference system with its control signal and system response being defined as follows:

$$\begin{aligned} \dot{x}_{ref}(t) &= A_m x_{ref}(t) + \\ &\quad b\Big(\omega u_{ref}(t) + \theta^\top(t) x_{ref}(t) + \sigma(t)\Big)\,, && (21) \\ u_{ref}(s) &= C(s)\frac{\bar{r}_{ref}(s)}{\omega}\,, \quad x_{ref}(0) = x_0, && (22) \\ y_{ref}(t) &= c^\top x_{ref}(t)\,, && (23) \end{aligned}$$

where $\bar{r}_{ref}(s)$ is the Laplace transformation of the signal

$$\bar{r}_{ref}(t) = -\theta^\top(t) x_{ref}(t) - \sigma(t) + k_g r(t)\,,$$

and k_g is introduced in (14). The next Lemma establishes stability of the closed-loop system in (21)-(23).

Lemma 5: If $D(s)$ verifies the condition in (19), the closed-loop reference system in (21)-(23) is stable.

Proof. Let

$$H(s) = (sI - A_m)^{-1} b\,. \tag{24}$$

It follows from (21)-(23) that

$$x_{ref}(s) = G(s) r_1(s) + H(s) C(s) k_g r(s)\,, \tag{25}$$

where $r_1(s)$ is the Laplace transformation of

$$r_1(t) = \theta^\top(t) x_{ref}(t) + \sigma(t) \tag{26}$$

with the following bound:

$$\|r_1\|_{\mathcal{L}_\infty} \leq L \|x_{ref}\|_{\mathcal{L}_\infty} + \|\sigma\|_{\mathcal{L}_\infty}\,. \tag{27}$$

Since $D(s)$ verifies the condition in (19), then Theorem 1, applied to (25), ensures that the closed-loop system in (21)-(23) is stable. □

Lemma 6: If $\theta(t)$ is constant, and $D(s) = 1/s$, then the closed-loop reference system in (21)-(23) is stable *iff* the matrix A_g in (20) is Hurwitz.

Proof. In case of constant $\theta(t)$, the state space form of the closed-loop system in (21)-(23) is given by:

$$\dot{x}_{ref}(t) = A_m x_{ref}(t) + b\left(\omega u_{ref}(t) + \theta^\top x_{ref}(t) + \sigma(t)\right), \tag{28}$$

$$\dot{u}_{ref}(t) = -\omega k u_{ref}(t) + k\left(-\theta^\top x_{ref}(t) - \sigma(t) + k_g r(t)\right), \tag{29}$$

$$y_{ref}(t) = c^\top x_{ref}(t)\,. \tag{30}$$

Letting

$$\zeta(t) = \begin{bmatrix} x_{ref}(t) \\ u_{ref}(t) \end{bmatrix},$$

it can be rewritten as

$$\dot{\zeta}(t) = A_g \zeta(t) + \begin{bmatrix} b\sigma(t) \\ -k\sigma(t) + k k_g r(t) \end{bmatrix}. \tag{31}$$

We note that the LTI system in (31) is stable iff A_g is Hurwitz, which concludes the proof. □

5.2 Bounded Error Signal

Lemma 7: For the system in (5) and the $\mathcal{L}_1$ adaptive controller in (8), (9)-(11) and (12), the tracking error between the system state and the companion model is bounded as follows:

$$\|\tilde{x}\|_{\mathcal{L}_\infty} \leq \sqrt{\frac{\theta_m}{\lambda_{\min}(P)\Gamma_c}}\,, \tag{32}$$

where

$$\begin{aligned} \theta_m \triangleq \max_{\theta\in\Theta} \sum_{i=1}^{n} 4\theta_i^2 + 4\Delta^2 + 4\left(\omega_u - \omega_l\right)^2 \\ +2\frac{\lambda_{\max}(P)}{\lambda_{\min}(Q)}\left(\max_{\theta\in\Theta}\|\theta\|_2 d_\theta + d_\sigma \Delta\right). \end{aligned} \tag{33}$$

Proof. Consider the following candidate Lyapunov function:

$$\begin{aligned} V(\tilde{x}(t), \tilde{\theta}(t), \tilde{\omega}(t), \tilde{\sigma}(t)) = \tilde{x}^\top(t) P \tilde{x}(t) + \\ \Gamma_c^{-1}\tilde{\theta}^\top(t)\tilde{\theta}(t) + \Gamma_c^{-1}\tilde{\omega}^2(t) + \Gamma_c^{-1}\tilde{\sigma}^2(t)\,, \end{aligned}$$

where

$$\tilde{\theta}(t) \triangleq \hat{\theta}(t) - \theta(t),\ \tilde{\sigma}(t) \triangleq \hat{\sigma}(t) - \sigma(t),\ \tilde{\omega}(t) \triangleq \hat{\omega}(t) - \omega\,. \tag{34}$$

It follows from (5) and (8) that

$$\dot{\tilde{x}}(t) = A_m\tilde{x}(t) + b\left(\tilde{\omega}(t)u(t) + \tilde{\theta}^\top(t)x(t) + \tilde{\sigma}(t)\right),\ \tilde{x}(0) = 0. \tag{35}$$

Using the projection based adaptation laws from (9)-(11), one has the following upper bound for $\dot{V}(t)$:

$$\dot{V}(t) \leq -\tilde{x}^\top(t)Q\tilde{x}(t) + \Gamma_c^{-1}\tilde{\theta}^\top(t)\dot{\theta}(t) + \Gamma_c^{-1}\tilde{\sigma}(t)\dot{\sigma}(t)\,. \tag{36}$$

The projection algorithm ensures that $\hat{\theta}(t) \in \Theta$, $\hat{\omega}(t) \in \Omega$, $\hat{\sigma}(t) \in \Delta$ for all $t \geq 0$, and therefore

$$\begin{aligned}\max_{t\geq 0}\left(\Gamma_c^{-1}\tilde{\theta}^\top(t)\tilde{\theta}(t) + \Gamma_c^{-1}\tilde{\omega}^2(t) + \Gamma_c^{-1}\tilde{\sigma}^2(t)\right) \leq \\ \left(\max_{\theta\in\Theta}\sum_{i=1}^{n}4\theta_i^2 + 4\Delta^2 + 4\left(\omega_u - \omega_l\right)^2\right)/\Gamma_c\end{aligned} \tag{37}$$

for any $t \geq 0$. If at any t

$$V(t) > \frac{\theta_m}{\Gamma_c}\,, \tag{38}$$

where θ_m is defined in (33), then it follows from (37) that

$$\tilde{x}^\top(t)P\tilde{x}(t) > 2\frac{\lambda_{\max}(P)}{\Gamma_c\lambda_{\min}(Q)}\left(\max_{\theta\in\Theta}\|\theta\|_2 d_\theta + d_\sigma\Delta\right), \tag{39}$$

and hence

$$\begin{aligned}\tilde{x}^\top(t)Q\tilde{x}(t) &> \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}\tilde{x}^\top(t)P\tilde{x}(t) \\ &> 2\frac{\max_{\theta\in\Theta}\|\theta\|_2 d_\theta + d_\sigma\Delta}{\Gamma_c}\,.\end{aligned}$$

The upper bounds in (7) along with the projection based adaptive laws lead to the following upper bound:

$$\frac{\tilde{\theta}^\top(t)\dot{\theta}(t) + \tilde{\sigma}(t)\dot{\sigma}(t)}{\Gamma_c} \leq 2\frac{\max_{\theta\in\Theta}\|\theta\|_2 d_\theta + d_\sigma\Delta}{\Gamma_c}\,. \tag{40}$$

Hence, if $V(t) > \dfrac{\theta_m}{\Gamma_c}$, then from (36) we have

$$\dot{V}(t) < 0\,. \tag{41}$$

Since we have set $\hat{x}(0) = x(0)$, we can verify that

$$V(0) \leq \Big(\max_{\theta\in\Theta} \sum_{i=1}^{n} 4\theta_i^2 + 4\Delta^2 + 4\left(\omega_u - \omega_l\right)^2 \Big) / \Gamma_c < \frac{\theta_m}{\Gamma_c}\,.$$

It follows from (41) that $V(t) \leq \dfrac{\theta_m}{\Gamma_c}$ for any $t \geq 0$. Since $\lambda_{\min}(P)\|\tilde{x}(t)\|^2 \leq \tilde{x}^\top(t)P\tilde{x}(t) \leq V(t)$, then

$$||\tilde{x}(t)||^2 \leq \frac{\theta_m}{\lambda_{\min}(P)\Gamma_c}\,,$$

which concludes the proof. □

Remark 1: We note that the bound in (32) is similar to the bounds derived in [18], assuming appropriate trajectory initialization to ensure transient performance improvement for system's output tracking. For the particular control architecture in this paper, the appropriate trajectory initialization is ensured by setting $\hat{x}(0) = x(0)$. However, due to the special filtering technique subject to $\mathcal{L}_1$-gain requirement, we obtain uniform smooth transient for systems's both signals, input and output, as proved in the next section.

5.3 Transient Performance

Let

$$H(s) = (sI - A_m)^{-1} b\,. \tag{42}$$

It follows from Lemma 4 that there exists $c_o \in \mathbb{R}^n$ such that

$$c_o^\top H(s) = \frac{N_n(s)}{N_d(s)}\,, \tag{43}$$

where the order of $N_d(s)$ is one more than the order of $N_n(s)$, and both $N_n(s)$ and $N_d(s)$ are stable polynomials.

Theorem 2: Given the system in (5) and the $\mathcal{L}_1$ adaptive controller defined via (8), (9)-(11) and (12) subject to (19), we have:

$$\|x - x_{ref}\|_{\mathcal{L}_\infty} \leq \gamma_1\,, \tag{44}$$

$$\|u - u_{ref}\|_{\mathcal{L}_\infty} \leq \gamma_2\,, \tag{45}$$

where

$$\gamma_1 = \frac{\|C(s)\|_{\mathcal{L}_1}}{1-\|H(s)(1-C(s))\|_{\mathcal{L}_1} L}\sqrt{\frac{\theta_m}{\lambda_{\max}(P)\Gamma_c}}, \tag{46}$$

$$\begin{aligned}\gamma_2 &= \left\|\frac{C(s)}{\omega}\right\|_{\mathcal{L}_1} L\gamma_1 + \\ &\quad \left\|\frac{C(s)}{\omega}\frac{1}{c_o^\top H(s)}c_o^\top\right\|_{\mathcal{L}_1}\sqrt{\frac{\theta_m}{\lambda_{\max}(P)\Gamma_c}}.\end{aligned} \tag{47}$$

Proof. Let

$$\begin{aligned}\tilde{r}(t) &= \tilde{\omega}(t)u(t) + \tilde{\theta}^\top(t)x(t) + \tilde{\sigma}(t), \\ r_2(t) &= \theta^\top(t)x(t) + \sigma(t).\end{aligned}$$

It follows from (12) that

$$\chi(s) = D(s)(\omega u(s) + r_2(s) - k_g r(s) + \tilde{r}(s)),$$

where $\tilde{r}(s)$ and $r_2(s)$ are the Laplace transformations of signals $\tilde{r}(t)$ and $r_2(t)$. Consequently

$$\chi(s) = \frac{D(s)}{1+k\omega D(s)}(r_2(s) - k_g r(s) + \tilde{r}(s)), \tag{48}$$

$$u(s) = -\frac{kD(s)}{1+k\omega D(s)}(r_2(s) - k_g r(s) + \tilde{r}(s)). \tag{49}$$

Using the definition of $C(s)$ from (15), we can write

$$\omega u(s) = -C(s)(r_2(s) - k_g r(s) + \tilde{r}(s)), \tag{50}$$

and the system in (5) consequently takes the form:

$$x(s) = H(s)\left((1-C(s))r_2(s) + C(s)k_g r(s) - C(s)\tilde{r}(s)\right). \tag{51}$$

It follows from (21)-(22) that

$$x_{ref}(s) = H(s)\left((1-C(s))r_1(s) + C(s)k_g r(s)\right), \tag{52}$$

where $r_1(s)$ is the Laplace transformation of the signal $r_1(t)$ defined in (26). Let $e(t) = x(t) - x_{ref}(t)$. Then, using (51), (52), one gets

$$e(s) = H(s)\left((1-C(s))r_3(s) - C(s)\tilde{r}(s)\right), e(0) = 0, \tag{53}$$

where $r_3(s)$ is the Laplace transformation of the signal

$$r_3(t) = \theta^\top(t)e(t)\,. \tag{54}$$

Lemma 7 gives the following upper bound:

$$\|e_t\|_{\mathcal{L}_\infty} \leq \|H(s)(1-C(s))\|_{\mathcal{L}_1}\|r_{3_t}\|_{\mathcal{L}_\infty} + \|r_{4_t}\|_{\mathcal{L}_\infty}\,, \tag{55}$$

where $r_4(t)$ is the signal with its Laplace transformation

$$r_4(s) = C(s)H(s)\tilde{r}(s).$$

From the relationship in (35) we have

$$\tilde{x}(s) = H(s)\tilde{r}(s)\,, \tag{56}$$

which leads to

$$r_4(s) = C(s)\tilde{x}(s)\,, \tag{57}$$

and hence

$$\|r_{4_t}\|_{\mathcal{L}_\infty} \leq \|C(s)\|_{\mathcal{L}_1}\|\tilde{x}_t\|_{\mathcal{L}_\infty}\,. \tag{58}$$

Using the definition of L in (18), one can verify easily that

$$\|(\theta^\top e)_t\|_{\mathcal{L}_\infty} \leq L\|e_t\|_{\mathcal{L}_\infty}\,, \tag{59}$$

and hence the following upper bound can be derived from (54):

$$\|r_{3_t}\|_{\mathcal{L}_\infty} \leq L\|e_t\|_{\mathcal{L}_\infty}\,. \tag{60}$$

From (55) we have

$$\|e_t\|_{\mathcal{L}_\infty} \leq \|H(s)(1-C(s))\|_{\mathcal{L}_1}L\|e_t\|_{\mathcal{L}_\infty} + \|C(s)\|_{\mathcal{L}_1}\|\tilde{x}_t\|_{\mathcal{L}_\infty}\,. \tag{61}$$

The upper bound from Lemma 7 and the $\mathcal{L}_1$-gain requirement from (19) lead to the following upper bound

$$\|e_t\|_{\mathcal{L}_\infty} \leq \frac{\|C(s)\|_{\mathcal{L}_1}}{1-\|H(s)(1-C(s))\|_{\mathcal{L}_1}L}\sqrt{\frac{\theta_m}{\lambda_{\max}(P)\Gamma_c}}\,, \tag{62}$$

which holds uniformly for all $t \geq 0$ and therefore leads to (44).

To prove the bound in (45), we notice that from (22) and (50) one can derive

$$u(s) - u_{ref}(s) = -\frac{C(s)}{\omega}\theta^\top(t)(x(s)-x_{ref}(s)) - r_5(s)\,, \tag{63}$$

where $r_5(s) = \frac{C(s)}{\omega}\tilde{r}(s)$. Therefore, it follows from Lemma 7 that

$$\|u - u_{ref}\|_{\mathcal{L}_\infty} \leq \frac{\|C(s)\|_{\mathcal{L}_1} L}{\omega} \|x - x_{ref}\|_{\mathcal{L}_\infty} + \|r_5\|_{\mathcal{L}_\infty} . \tag{64}$$

We have

$$\begin{aligned} r_5(s) &= \frac{C(s)}{\omega} \frac{1}{c_o^\top H(s)} c_o^\top H(s) \tilde{r}(s) \\ &= \frac{C(s)}{\omega} \frac{1}{c_o^\top H(s)} c_o^\top \tilde{x}(s) , \end{aligned}$$

where c_o is introduced in (43). Using the polynomials from (43), we can write that

$$\frac{C(s)}{\omega} \frac{1}{c_o^\top H(s)} = \frac{C(s)}{\omega} \frac{N_d(s)}{N_n(s)} ,$$

where $N_d(s)$, $N_n(s)$ are stable polynomials and the order of $N_n(s)$ is one less than the order of $N_d(s)$. Since $C(s)$ is stable and strictly proper, the complete system $C(s)\frac{1}{c_o^\top H(s)}$ is proper and stable, which implies that its $\mathcal{L}_1$ gain exists and is finite. Hence, we have

$$\|r_5\|_{\mathcal{L}_\infty} \leq \left\| \frac{C(s)}{\omega} \frac{1}{c_o^\top H(s)} c_o^\top \right\|_{\mathcal{L}_1} \|\tilde{x}\|_{\mathcal{L}_\infty} .$$

Lemma 7 consequently leads to the upper bound:

$$\|r_5\|_{\mathcal{L}_\infty} \leq \left\| \frac{C(s)}{\omega} \frac{1}{c_o^\top H(s)} c_o^\top \right\|_{\mathcal{L}_1} \sqrt{\frac{\theta_m}{\lambda_{\max}(P)\Gamma_c}} ,$$

which, when substituted into (64), leads to (45). □

Theorem 3: For the closed-loop system in (5) with $\mathcal{L}_1$ adaptive controller defined via (8), (9)-(11) and (12), subject to (20), if $\theta(t)$ is (unknown) constant and $D(s) = \dfrac{1}{s}$, we have:

$$\|x - x_{ref}\|_{\mathcal{L}_\infty} \leq \gamma_3 , \tag{65}$$

$$\|u - u_{ref}\|_{\mathcal{L}_\infty} \leq \gamma_4 , \tag{66}$$

where

$$\gamma_3 = \left\| H_g(s) C(s) \frac{1}{c_o^\top H(s)} c_o^\top \right\|_{\mathcal{L}_1} \sqrt{\frac{\theta_m}{\lambda_{\max}(P)\Gamma_c}}, \tag{67}$$

$$\begin{aligned} \gamma_4 = & \left\| \frac{C(s)}{\omega} \theta^\top \right\|_{\mathcal{L}_1} \gamma_3 + \\ & \left\| \frac{C(s)}{\omega} \frac{1}{c_o^\top H(s)} c_o^\top \right\|_{\mathcal{L}_1} \sqrt{\frac{\theta_m}{\lambda_{\max}(P)\Gamma_c}} , \end{aligned} \tag{68}$$

and

$$H_g(s) = (sI - A_g)\begin{bmatrix} b \\ 0 \end{bmatrix}.$$

Proof. Recall that for constant θ we had

$$D(s) = \frac{1}{s}, \quad C(s) = \frac{k\omega}{s + k\omega}.$$

Let

$$\zeta(s) = -\frac{C(s)}{\omega}\theta^\top e(s)\,.$$

With this notation, (53) can be written as

$$e(s) = H(s)\left(\theta^\top e(s) + \omega\zeta(s) - C(s)\tilde{r}(s)\right)$$

and further put into state space form as:

$$\begin{bmatrix} \dot{e}(t) \\ \dot{\zeta}(t) \end{bmatrix} = A_g \begin{bmatrix} e(t) \\ \zeta(t) \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} r_6(t)\,, \tag{69}$$

where $r_6(t)$ is the signal with its Laplace transformation

$$r_6(s) = -C(s)\tilde{r}(s)\,. \tag{70}$$

Let

$$x_\zeta(t) = [e^\top(t)\ \zeta(t)]^\top.$$

Since A_g is Hurwitz, then $H_g(s)$ is stable and strictly proper. It follows from (69) that

$$x_\zeta(s) = -H_g(s)C(s)\tilde{r}(s)\,.$$

Therefore, we have

$$\begin{aligned} x_\zeta(s) &= -H_g(s)C(s)\frac{1}{c_o^\top H(s)}c_o^\top H(s)\tilde{r}(s) \\ &= -H_g(s)C(s)\frac{1}{c_o^\top H(s)}c_o^\top \tilde{x}(s)\,, \end{aligned}$$

where c_o is introduced in (43). It follows from (43) that $H_g(s)C(s)\frac{1}{c_o^\top H(s)} = H_g(s)C(s)\frac{N_d(s)}{N_n(s)}$, where $N_d(s)$, $N_n(s)$ are stable polynomials and the order of $N_n(s)$ is one less than the order of $N_d(s)$. Since both $H_g(s)$ and $C(s)$ are stable

and strictly proper, the complete system $H_g(s)C(s)\frac{1}{c_o^\top H(s)}$ is proper and stable, which implies that its $\mathcal{L}_1$ gain exists and is finite. Hence, we have

$$\|x_\zeta\|_{\mathcal{L}_\infty} \le \left\| H_g(s)C(s)\frac{1}{c_o^\top H(s)} c_o^\top \right\|_{\mathcal{L}_1} \|\tilde{x}\|_{\mathcal{L}_\infty} .$$

The proof of (66) is similar to the proof of (45). □

Corollary 2: Given the system in (5) and the $\mathcal{L}_1$ adaptive controller defined via (8), (9)-(11) and (12) subject to (19), we have:

$$\lim_{\Gamma_c\to\infty} (x(t) - x_{ref}(t)) = 0\,, \qquad \forall t \ge 0, \tag{71}$$

$$\lim_{\Gamma_c\to\infty} (u(t) - u_{ref}(t)) = 0\,, \qquad \forall t \ge 0\,. \tag{72}$$

Thus, the tracking error between $x(t)$ and $x_{ref}(t)$, as well between $u(t)$ and $u_{ref}(t)$, is uniformly bounded by a constant inverse proportional to Γ_c. This implies that during the transient one can achieve arbitrarily close tracking performance for both signals simultaneously by increasing Γ_c.

5.4 Asymptotic Convergence

Since the bounds in (44) and (45) are uniform for all $t \ge 0$, they are in charge for both transient and steady state performance. In case of constant θ one can prove in addition the following asymptotic result.

Lemma 8: Given the system in (5) with constant θ and $\mathcal{L}_1$ adaptive controller defined via (8), (9)-(11) and (12) subject to (19), we have:

$$\lim_{t\to\infty} \tilde{x}(t) = 0\,. \tag{73}$$

Proof: It follows from Lemmas 5 and 7, and Theorem 2 that both $x(t)$ and $\hat{x}(t)$ in $\mathcal{L}_1$ adaptive controller are bounded for bounded reference inputs. The adaptive laws in (9)-(11) ensure that the estimates $\hat{\theta}(t)$, $\hat{\omega}(t)$, $\hat{\sigma}(t)$ are also bounded. Hence, it can be checked easily from (35) that $\dot{\tilde{x}}(t)$ is bounded, and it follows from Barbalat's lemma that $\lim_{t\to\infty} \tilde{x}(t) = 0$. □

5.5 Design Guidelines

We note that the control law $u_{ref}(t)$ in the closed-loop reference system, which is used in the analysis of $\mathcal{L}_\infty$ norm bounds, is not implementable since its definition involves the unknown parameters. Theorem 2 ensures that the $\mathcal{L}_1$ adaptive

controller approximates $u_{ref}(t)$ both in transient and steady state. So, it is important to understand how these bounds can be used for ensuring uniform transient response with *desired* specifications. We notice that the following *ideal* control signal

$$u_{ideal}(t) = \frac{k_g r(t) - \theta^\top(t) x_{ref}(t) - \sigma(t)}{\omega} \tag{74}$$

is the one that leads to desired system response:

$$\begin{aligned} \dot{x}_{ref}(t) &= A_m x_{ref}(t) + b k_g r(t) && (75) \\ y_{ref}(t) &= c^\top x_{ref}(t) && (76) \end{aligned}$$

by cancelling the uncertainties exactly. In the closed-loop reference system (21)-(23), $u_{ideal}(t)$ is further low-pass filtered by $C(s)$ in (22) to have guaranteed low-frequency range. Thus, the reference system in (21)-(23) has a different response as compared to (75), (76) with (74). In [21], specific design guidelines are suggested for selection of $C(s)$ to ensure that in case of constant θ the response of (28), (29), (30) can be made as close as possible to (75), (76) with (74). In case of fast varying $\theta(t)$, it is obvious that the bandwidth of the controller needs to be matched correspondingly.

6 Simulations

As an illustrative example, consider a single-link robot arm which is rotating on a vertical plane. The system dynamics are given by:

$$I\ddot{q}(t) + \frac{MgL\cos q(t)}{2} + F(t)\dot{q}(t) + F_1(t)q(t) + \bar{\sigma}(t) = u(t)\,, \tag{77}$$

where $q(t)$ and $\dot{q}(t)$ are measured angular position and velocity, respectively, $u(t)$ is the input torque, I is the unknown moment of inertia, M is the unknown mass, , L is the unknown length, $F(t)$ is an unknown time-varying friction coefficient, $F_1(t)$ is position dependent external torque, and $\bar{\sigma}(t)$ is unknown bounded disturbance. The control objective is to design $u(t)$ to achieve tracking of bounded reference input $r(t)$ by $q(t)$. Let

$$x = [q \quad \dot{q}]^\top .$$

The system in (77) can be presented in the state-space form as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + b\Big(\frac{u(t)}{I} + \frac{MgL\cos(x_1(t))}{2I} + \frac{\bar{\sigma}(t)}{I} \\ &\quad + \frac{F_1(t)}{I}x_1(t) + \frac{F(t)}{I}x_2(t)\Big), \quad x(0) = x_0\,, \\ y(t) &= c^\top x(t)\,, && (78) \end{aligned}$$

where x_0 is the initial condition,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad c = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{79}$$

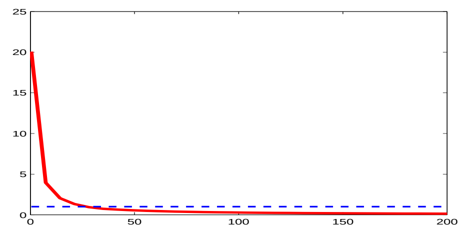


Fig. 3: $\|G(s)\|_{\mathcal{L}_1} L$ with respect to ωk.

The system can be further put into the form:

$$\begin{aligned} \dot{x}(t) &= A_m x(t) + b(\omega u(t) + \theta^\top(t) x(t) + \sigma(t)), \\ y(t) &= c^\top x(t), \quad x(0) = x_0, \end{aligned}$$

where $\omega = \frac{1}{I}$ is the unknown control effectiveness,

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -1.4 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, c = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{80}$$

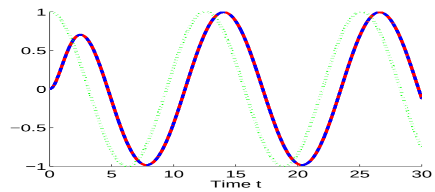
$$\begin{aligned} \theta(t) &= \left[1 + \frac{F_1(t)}{I} \quad 1.4 + \frac{F(t)}{I} \right]^\top, \\ \sigma(t) &= \frac{MgL\cos(x_1(t))}{2I} + \frac{\bar{\sigma}(t)}{I}. \end{aligned}$$

Let the unknown control effectiveness, time-varying parameters and disturbance be given by:

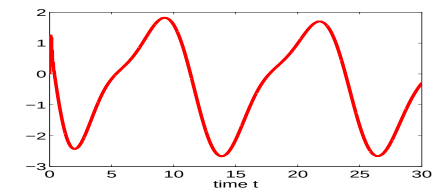
$$\begin{aligned} \omega &= 1, \\ \theta(t) &= [2 + \cos(\pi t) \quad 2 + 0.3\sin(\pi t) + 0.2\cos(2t)]^\top, \\ \sigma(t) &= \sin(\pi t), \end{aligned} \tag{81}$$

so that the compact sets can be conservatively chosen as

$$\Omega = [0.2,\ 5],\ \Theta = [-10,\ 10],\ \Delta = [-10,\ 10]. \tag{82}$$



(a) $x_1(t)$ (solid), $\hat{x}_1(t)$ (dashed), and $r(t)$(dotted)



(b) Time-history of $u(t)$

Fig. 4: Performance of $\mathcal{L}_1$ adaptive controller for $\sigma(t) = \sin(\pi t)$

For implementation of the $\mathcal{L}_1$ adaptive controller (8), (9)-(11) and (12), we need to verify the $\mathcal{L}_1$ stability requirement in (19). Letting

$$D(s) = 1/s\,,$$

we have

$$G(s) = \frac{\omega k}{s + \omega k} H(s), \tag{83}$$

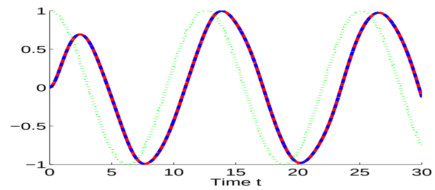
where

$$H(s) = \begin{bmatrix} \frac{1}{s^2+1.4s+1} \\ \frac{s}{s^2+1.4s+1} \end{bmatrix}. \tag{84}$$

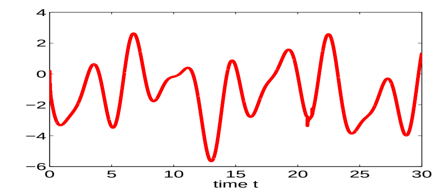
We can check easily that for our selection of compact sets in (82), the resulting $L = 20$ in (18). In Fig. 6, we plot $\|G(s)\|_{\mathcal{L}_1} L$ as a function of ωk and compare it to 1. We notice that for $\omega k > 30$, we have $\|G(s)\|_{\mathcal{L}_1} L < 1$. Since $\omega > 0.5$, we set $k = 60$. At last, we set the adaptive gain as $\Gamma_c = 10000$.

The simulation results of the $\mathcal{L}_1$ adaptive controller are shown in Figures 4(a)-4(b) for reference input $r = \cos(\pi t)$. Next, we consider different disturbance signal:

$$\sigma(t) = \cos(x_1(t)) + 2\sin(10t) + \cos(15t)\,.$$



(a) $x_1(t)$ (solid), $\hat{x}_1(t)$ (dashed), and $r(t)$(dotted)



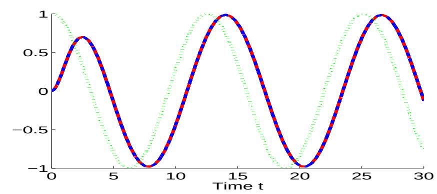
(b) Time-history of $u(t)$

Fig. 5: Performance of $\mathcal{L}_1$ adaptive controller for $\sigma(t) = \cos(x_1(t)) + 2\sin(10t) + \cos(15t)$

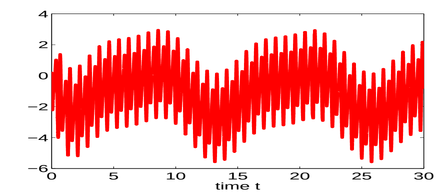
The simulation results are shown in 5(a)-5(b). Finally, we consider much higher frequencies in the disturbance:

$$\sigma(t) = \cos(x_1(t)) + 2\sin(100t) + \cos(150t)\,.$$

The simulation results are shown in 6(a)-6(b). We note that the $\mathcal{L}_1$ adaptive controller guarantees smooth and uniform transient performance in the presence of different unknown nonlinearities and time-varying disturbances. The controller frequencies are exactly matched with the frequencies of the disturbance that it is supposed to cancel out. We also notice that $x_1(t)$ and $\hat{x}_1(t)$ are almost the same in Figs. 4(a), 5(a) and 6(a).



(a) $x_1(t)$ (solid), $\hat{x}_1(t)$ (dashed), and $r(t)$(dotted)



(b) Time-history of $u(t)$

Fig. 6: Performance of $\mathcal{L}_1$ adaptive controller for $\sigma(t) = \cos(x_1(t)) + 2\sin(100t) + \cos(150t)$

7 Conclusion

A novel $\mathcal{L}_1$ adaptive control architecture is presented that has guaranteed transient response in addition to stable tracking for systems with time-varying unknown parameters and bounded disturbances. The control signal and the system response approximate the same signals of a closed-loop reference LTI system, which can be designed to achieve desired specifications. In Part II of this paper [22], we derive the stability margins of this $\mathcal{L}_1$ adaptive control architecture.

References

[1] A. Datta and M.-T. Ho. On modifying model reference adaptive control schemes for performance improvement. *IEEE Trans. Autom. Contr.*, 39(9):1977–1980, 1994.

[2] G. Bartolini, A. Ferrara, and A. A. Stotsky. Robustness and performance of an indirect adaptive control scheme in presence of bounded disturbances. *IEEE Trans. Autom. Contr.*, 44(4):789–793, 1999.

[3] J. Sun. A modified model reference adaptive control scheme for improved transient performance. *IEEE Trans. Autom. Contr.*, 38(7):1255–1259, 1993.

[4] D.E. Miller and E.J. Davison. Adaptive control which provides an arbitrarily good transient and steady-state response. *IEEE Trans. Autom. Contr.*, 36(1):68–81, 1991.

[5] R. Costa. Improving transient behavior of model-reference adaptive control. *In Proc. of American Control Conference*, pp. 576–580, 1999.

[6] B.E. Ydstie. Transient performance and robustness of direct adaptive control. *IEEE Trans. Autom. Contr.*, 37(8):1091–1105, 1992.

[7] M. Krstic, P. V. Kokotovic, and I. Kanellakopoulos. Transient performance improvement with a new class of adaptive controllers. *Systems & Control Letters*, 21:451–461, 1993.

[8] R. Ortega. Morse's new adaptive controller: Parameter convergence and transient performance. *IEEE Trans. Autom. Contr.*, 38(8):1191–1202, 1993.

[9] Z. Zang and R. Bitmead. Transient bounds for adaptive control systems. *Proc. of* 30^{th} *IEEE Conference on Decision and Control*, pp. 2724–2729, 1990.

[10] A. Datta and P. Ioannou. Performance analysis and improvement in model reference adaptive control. *IEEE Trans. Autom. Contr.*, 39(12):2370–2387, 1994.

[11] A. M. Arteaga and Y. Tang. Adaptive control of robots with an improved transient performance. *IEEE Trans. Autom. Contr.*, 47(7):1198–1202, 2002.

[12] K. S. Narendra and J. Balakrishnan. Improving transient response of adaptive control systems using multiple models and switching. *IEEE Trans. Autom. Contr.*, 39(9):1861–1866, 1994.

[13] R. Marino and P. Tomei. Adaptive tracking of linear systems with arbitrarily time-varying parameters. *In Proc. of the* 38^{th} *IEEE Conference on Decision and Control*, pp. 4784-4789, 1999.

[14] R. Marino and P. Tomei. An adaptive output feedback control for a class of nonlinear systems with time-varying parameters. *IEEE Trans. Autom. Contr.*, 44(11):2190–2194, 1999.

[15] K.S. Tsakalis and P.A. Ioannou. Adaptive control of linear time-varying plants. *Automatica*, 23:459–468, 1987.

[16] R. H. Middleton and G. C. Goodwin. Adaptive control of time-varying linear systems. *IEEE Trans. Autom. Contr.*, 33(1):150–155, 1988.

[17] Y. Zang and P. A. Ioannou. Adaptive control of linear time-varying systems. *In Proc. of IEEE 35^{th} Conference on Decision and Control*, pages 837–842, 1996.

[18] M. Krstic, I. Kanellakopoulos, and P. Kokotovic. *Nonlinear and Adaptive Control Design*. John Wiley & Sons, New York, 1995.

[19] C. Cao, V. V. Patel, N. Hovakimyan, and E. Lavretsky. Effect of high-gain on the phase margin in four different control schemes. *In proc. of AIAA Guidance, Navigation and Control Conference*, 2006.

[20] C. Cao and N. Hovakimyan. Design and analysis of a novel $\mathcal{L}_1$ adatpive control architecture with guaranteed transient performance, Part I: Control signal and asymptotic stability. *In Proc. of American Control Conference*, 2006.

[21] C. Cao and N. Hovakimyan. Design and analysis of a novel $\mathcal{L}_1$ adatpive control architecture with guaranteed transient performance, Part II: Guaranteed transient performance. *In Proc. of American Control Conference*, 2006.

[22] C. Cao and N. Hovakimyan. Stability margins of $\mathcal{L}_1$ adaptive controller: Part II. *Submitted to IEEE Conference on Decision and Control*, 2006.

[23] P. Ioannou and J. Sun. *Robust Adaptive Control*. Prentice Hall, 1996.

[24] H. K. Khalil. *Nonlinear Systems*. Prentice Hall, Englewood Cliffs, NJ, 2002.

[25] K. Zhou and J. C. Doyle. *Essentials of Robust Control*. Prentice Hall, Englewood Cliffs, NJ, 1998.